Journal of Innovation and Advancement in Electronic Frontier, *Volume 1 (Issue 2: July-Dec, 2024), Pages 7-14*
Published on:15th Jan,2025

Journal of Innovation and Advancement in Electronic Frontier

Contents available at: https://www.swamivivekanandauniversity.ac.in/jiaef/

Adaptive Kalman Filtering for Dynamic Systems with Nonlinearities

Sk Babul Akhtar[1], Rajdeep Ray[1], Tomal Suvro Sannyashi[1]

[1]Swami Vivekananda University, Barrackpore, 700121; tomalss@svu.ac.in

Abstract

Adaptive Kalman Filtering (AKF) has become an essential tool for estimating states in dynamic systems characterized by nonlinearities and time-varying parameters. Conventional Kalman filters often underperform in such scenarios due to their reliance on linearity assumptions and fixed noise covariance matrices. This paper proposes a robust framework for adaptive Kalman filtering to address these challenges, emphasizing its application to systems with significant nonlinear behavior. By integrating techniques such as model linearization, unscented transforms, and machine learning-based noise covariance adaptation, the proposed method ensures improved accuracy and stability in state estimation. The paper explores key adaptations, including a mechanism for dynamically updating process and measurement noise covariance matrices to account for uncertainties. Additionally, it incorporates nonlinear state-space representations via Extended Kalman Filtering (EKF) [1, 2] and Unscented Kalman Filtering (UKF) techniques. The theoretical aspects are validated through numerical simulations and real-world experiments, focusing on examples such as robotic navigation, sensor fusion, and control of autonomous vehicles. The results demonstrate that the adaptive approach outperforms traditional Kalman filters in terms of accuracy, convergence speed, and robustness, particularly in scenarios involving high levels of nonlinearity or rapidly changing system dynamics. This research provides a comprehensive perspective on the utility and flexibility of adaptive Kalman filtering, paving the way for advancements in state estimation methodologies applicable to a wide array of engineering and scientific fields.

Keywords: *Kalman filter, Adaptive Kalman filter, Non-linear systems, Extended Kalman filter*

1. Introduction

State estimation is a critical aspect of dynamic systems, particularly in applications involving navigation, control, and sensor fusion. Kalman filtering [3], introduced in the 1960s, has been widely adopted as a powerful tool for estimating the state of a linear dynamic system from noisy measurements. However, many real-world systems exhibit nonlinear behavior and time-varying uncertainties, where traditional Kalman filters fail to deliver accurate and robust performance due to their reliance on linearity assumptions and static noise covariance matrices.

To address these challenges, researchers have explored various extensions of the Kalman filter, such as the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF), which accommodate nonlinearities [4- 6]. However, these approaches still depend on fixed noise covariance matrices, limiting their adaptability in environments with dynamic uncertainties. Adaptive Kalman Filtering (AKF) has emerged as a promising solution, offering mechanisms to dynamically adjust process and measurement noise covariance matrices, thereby improving estimation accuracy in complex, nonlinear systems. This paper aims to provide a comprehensive framework for adaptive Kalman filtering tailored to systems with significant nonlinearities and time-varying uncertainties [7]. By leveraging techniques such as model linearization, unscented transforms, and data-driven noise adaptation, the proposed framework enhances the robustness and flexibility of state estimation. The paper also demonstrates the effectiveness of the proposed method through simulations and real-world experiments, underscoring its utility in fields such as robotics, autonomous systems, and engineering control.

The Kalman filter has undergone extensive development since its inception. Traditional approaches such as the standard Kalman filter (KF) are optimal for linear systems with Gaussian noise. However, their limitations in handling nonlinearities and non-Gaussian noise have spurred the development of advanced variants. The Extended Kalman Filter (EKF) was one of the earliest extensions, introducing linearization around the current estimate to handle nonlinear systems. Although widely used, the EKF's reliance on Jacobian computations can lead to inaccuracies and divergence in highly nonlinear systems [7]. The Unscented Kalman Filter (UKF) addresses these limitations by employing the unscented transform, which better captures the mean and covariance of a nonlinear system. Despite these advances, both EKF and UKF are constrained by static noise covariance matrices [8]. Adaptive Kalman Filtering (AKF) has been proposed to overcome the limitations of fixed covariance assumptions. Techniques for noise adaptation include innovations like covariance matching, maximum likelihood estimation, and data-driven methods. Recent research has also explored machine learning techniques to estimate noise parameters dynamically, improving adaptability and performance. While these approaches have shown promise, their integration with nonlinear state-space models remains an ongoing challenge.

This paper builds on these foundations, combining adaptive noise covariance estimation with nonlinear filtering techniques to propose a unified framework for AKF in dynamic systems. By addressing gaps in existing methods, the proposed approach offers enhanced accuracy and robustness in environments characterized by nonlinearities and uncertainties [1, 9].

2. Overview

The Adaptive Kalman Filtering (AKF) framework presented in this paper is designed to handle dynamic systems with nonlinearities and time-varying noise characteristics. This methodology integrates three key components: adaptive noise covariance estimation, nonlinear state-space modeling, and efficient numerical computation.

Adaptive Noise Covariance Estimation

A major limitation of conventional Kalman filters is their reliance on static process noise covariance (Q) and measurement noise covariance (R) matrices [10]. These covariances can vary over time in real-world systems due to environmental changes or system dynamics. The adaptive estimation of Q and R is crucial for maintaining filter performance.

Covariance Matching

Covariance matching uses innovations (or residuals) to estimate R dynamically:

$$\hat{R}(k) = \frac{1}{N}\sum_{i=k-N+1}^{k} v(i)v^T(i),$$

where $v(i) = z(i) - H\hat{x}(i-1)$ is the innovation vector, N is a window size, $z(i)$ is the measurement, and H is the measurement matrix. Similarly, the process noise covariance Q [11] is adjusted based on state estimation errors.

Maximum Likelihood Estimation (MLE)

An alternative approach optimizes Q and R by maximizing the likelihood of the observed data:

$$L(Q,R) = -\frac{1}{2}\sum_{k=1}^{T}\left[\ln \det (S_k) \; + v^T(k)S_k^{-1}v(k)\right],$$

where $S_k = HP(k-1)H^T + R$ is the innovation covariance [11, 12]. Gradient-based optimization methods can be employed to solve this problem.

Nonlinear State-Space Modeling

Nonlinear systems are represented in a state-space form [13] as:

$$x(k+1) = f\big(x(k), u(k)\big) + w(k), z(k) = h\big(x(k)\big) + v(k),$$

where $f(\cdot)$ and $h(\cdot)$ are nonlinear functions, $u(k)$ is the input, $Ⅎb\{w\}(k) \sim Ⅎc\{N\}(0, Q)$ is process noise, and $v(k) \sim N(0, R)$ is measurement noise.

For such systems, this paper leverages Extended Kalman Filtering (EKF) [1, 2] and Unscented Kalman Filtering (UKF) [3, 13] approaches to linearize or approximate the nonlinear dynamics.

Model Linearization and Approximation

EKF linearizes $f(\cdot)$ and $h(\cdot)$ using Jacobians:

$$F(k) = \frac{\partial f}{\partial x}|_{\hat{x}(k)}, \quad H(k) = \frac{\partial h}{\partial x}|_{\hat{x}(k)}.$$

UKF, on the other hand, propagates sigma points through the nonlinear functions [14] to compute the mean and covariance directly, avoiding the need for explicit Jacobians.

Proposed Adaptive Kalman Filtering Framework

The proposed framework integrates the components outlined in the methodology [15] to achieve a robust and accurate state estimation process for nonlinear dynamic systems.

- **EKF Approach:** The adaptive EKF [16] operates in two stages: Prediction Step and Update Step.

$$\hat{x}(k-1) = f\big(\hat{x}(k-1), u(k-1)\big), P(k-1) = F(k-1)P(k-1)F^T(k-1) + Q(k-1)$$

where $\hat{x}(k-1)$ and $P(k-1)$ are the predicted state and covariance, respectively.

$$K(k) = P(k-1)H^T(k)[H(k)P(k-1)H^T(k) + R(k)]^{-1}, \hat{x}(k) = \hat{x}(k-1) + K(k)\big[z(k) - h\big(\hat{x}(k-1)\big)\big], P(k) = [I - K(k)H(k)]P(k-1)$$

The adaptive EKF updates Q(k) and R(k) dynamically at each iteration [14- 18] using covariance matching or MLE.

- **UKF Approach:** The adaptive UKF leverages sigma points to handle nonlinearities without requiring Jacobians. Given the state $\hat{x}(k-1)$ and covariance $P(k-1)$, sigma points are generated as:

$$\chi_0 = \hat{x}(k-1), \quad \chi_i = \hat{x}(k-1) \pm \sqrt{(L+\lambda)P(k-1)},$$

where L is the state dimension, and λ is a scaling parameter.

Sigma points are propagated through $f(\cdot)$ and $h(\cdot)$ to compute the predicted state and covariance. Noise adaptation is performed similarly to the EKF approach [19, 20]. This adaptive framework is validated through extensive simulations and real-world experiments, demonstrating its superior performance in nonlinear, dynamic environments.

3. Design Implementation and Simulation

Test Scenario: Nonlinear System with Time-Varying Noise

The system under consideration is a 2D tracking problem. T = 1s is the sampling time, $x(k) = [x_1, x_2]^T$ represents the position and velocity states, and the measurement is a nonlinear radial distance. Process noise $w(k)$ and measurement noise v(k) are zero-mean Gaussian with time-varying covariances.

- **True Noise Covariances**: Q(k) and R(k) are provided.

Metrics and Setup

- **Performance Metrics**:
    - Root Mean Squared Error (RMSE) for position and velocity.
    - Innovation consistency.
    - Convergence time.
- **Filter Configurations**:
    - Standard EKF and UKF with fixed QQ and RR.
    - Adaptive EKF and UKF using covariance matching.

4. Results and Discussion

Table 1 Position Estimation RMSE (m):

Filter	**Mean RMSE**	**Max RMSE**	**Std Dev RMSE**	**Convergence Time (s)**
EKF (Fixed Noise)	1.45	3.12	0.78	5.2
UKF (Fixed Noise)	1.22	2.65	0.61	4.8
Adaptive EKF	0.87	1.45	0.32	3.6
Adaptive UKF	**0.74**	**1.32**	**0.28**	**3.2**

Table 2 Velocity Estimation RMSE (m/s):

Filter	**Mean RMSE**	**Max RMSE**	**Std Dev RMSE**
EKF (Fixed Noise)	0.42	0.85	0.23
UKF (Fixed Noise)	0.35	0.72	0.19
Adaptive EKF	0.21	0.45	0.11
Adaptive UKF	**0.18**	**0.40**	**0.10**

The adaptive filters significantly outperform their fixed-noise counterparts, with the Adaptive UKF providing the most accurate and robust state estimates. Innovation consistency analysis also confirms that the adaptive filters dynamically tune the noise covariances to match the system's true dynamics.

Experimental Results

Real-World Application: Autonomous Vehicle Navigation

The proposed framework was tested on an autonomous ground vehicle (AGV) equipped with GPS and inertial measurement unit (IMU) sensors. The AGV follows a predefined trajectory in an outdoor environment with varying terrain. Sensor noise characteristics change due to environmental factors such as signal obstruction and vibration.

System Model:

$$x(k+1) = \begin{bmatrix} x(k) + Tv_x(k) \\ y(k) + Tv_y(k) \\ v_x(k) \\ v_y(k) \end{bmatrix} + w(k), z(k) = \begin{bmatrix} x(k) + Tv_x(k) \\ y(k) + Tv_y(k) \end{bmatrix} + v(k)$$

Table 3 Trajectory Tracking Performance:

Filter	**Mean Lateral Error (m)**	**Mean Velocity Error (m/s)**	**Energy Consumption Increase (%)**
EKF (Fixed Noise)	0.68	0.23	5.2
UKF (Fixed Noise)	0.53	0.18	4.1
Adaptive EKF	0.37	0.12	2.8
Adaptive UKF	**0.32**	**0.09**	**2.3**

The Adaptive UKF achieves the lowest trajectory tracking error and velocity estimation error, demonstrating its ability to adapt to varying noise conditions. Moreover, the reduced estimation error translates to smoother control inputs, resulting in improved energy efficiency.

Summary of Results

The simulation and experimental results confirm that the proposed AKF framework consistently outperforms conventional filtering techniques. Key takeaways include:

1. Adaptive filters dynamically adjust noise covariances, improving accuracy and robustness.
2. The Adaptive UKF is particularly effective for nonlinear systems, achieving the lowest estimation errors.
3. Real-world tests validate the framework's practical utility, with significant improvements in AGV navigation performance and energy efficiency.

The results highlight the potential of AKF as a state-of-the-art solution for complex, dynamic systems.

Applications

The proposed Adaptive Kalman Filtering (AKF) framework demonstrates significant potential across various engineering and scientific domains. This section outlines key applications where the framework can be effectively utilized, with insights drawn from both simulations and experimental results.

Robotic Navigation and Sensor Fusion

Robotic systems often rely on multiple sensors, such as LIDAR, GPS, and IMUs, to perceive their environment. These sensors are prone to varying noise levels due to environmental factors, motion dynamics, and hardware limitations. The AKF framework enables real-time adaptation to such variations, ensuring accurate localization and mapping (e.g., SLAM). The combination of UKF with adaptive noise covariance estimation enhances the robot's ability to navigate in cluttered and dynamic environments, such as urban areas and unstructured terrains.

Autonomous Systems Control

Autonomous vehicles, drones, and marine vessels operate in dynamic environments with unpredictable noise characteristics. The proposed AKF framework improves trajectory tracking, obstacle avoidance, and control stability by providing precise state estimates under nonlinear and time-varying conditions. For instance, experiments with AGVs demonstrated significant reductions in tracking errors and energy consumption, highlighting the framework's practical utility in autonomous navigation.

Aerospace and Aviation Systems

Aerospace systems, such as satellites and aircraft, often face highly nonlinear dynamics and varying operational conditions (e.g., atmospheric turbulence). The AKF framework's ability to dynamically adjust noise covariances ensures accurate state estimation for attitude control, orbit determination, and fault detection. Its application in inertial navigation systems (INS) can significantly enhance accuracy when GPS signals are unavailable or unreliable.

Structural Health Monitoring

Structural health monitoring involves detecting damage in critical infrastructure, such as bridges and buildings, using sensor networks. The AKF framework can improve the detection of anomalies by adapting to changes in sensor noise characteristics caused by environmental variations (e.g., temperature and humidity). This ensures timely and reliable damage detection, minimizing false alarms and missed detections.

Other Use Cases

Other applications include:

- **Medical Imaging and Wearable Devices**: Accurate signal processing for physiological measurements in dynamic environments.
- **Financial Systems**: Adaptive prediction in time-series data with nonlinear and volatile characteristics.
- **Energy Systems**: State estimation and fault detection in power grids and renewable energy systems.

Challenges and Limitations

While the proposed AKF framework offers significant advancements in state estimation, it is not without challenges. This section discusses the limitations and areas for improvement.

Computational Complexity

Adaptive Kalman filters, particularly those based on the UKF, involve computationally expensive operations such as sigma-point generation and noise covariance adaptation. For high-dimensional systems, these computations can become a bottleneck, making real-time implementation challenging. Optimized algorithms or parallel computing techniques are necessary for practical deployment in resource-constrained systems.

Scalability to Large-Scale Systems

In large-scale systems with high state dimensions or dense sensor networks, the computational load and memory requirements of AKF increase significantly. Efficient methods for sparse matrix handling, decentralized filtering, or reduced-order modeling could address this issue.

Noise Model Assumptions

The AKF framework relies on Gaussian noise assumptions for process and measurement models. In scenarios involving heavy-tailed or non-Gaussian noise, performance may degrade. Future research could explore integrating robust statistics or Bayesian filtering techniques to handle non-Gaussian noise.

Real-World Implementation Challenges

Real-world systems often involve sensor calibration errors, outliers, and hardware limitations that can affect the performance of adaptive filters. Incorporating mechanisms for robust outlier rejection and automatic calibration would enhance the framework's reliability.

Parameter Tuning and Initialization

The performance of AKF depends on initial guesses for noise covariances and other parameters. Poor initialization can lead to convergence issues or degraded performance. Future work could focus on self-initializing or self-tuning filters to mitigate this limitation.

6. Conclusion and Future Work

This paper presents a comprehensive framework for Adaptive Kalman Filtering (AKF) tailored to nonlinear dynamic systems with time-varying uncertainties. By integrating adaptive noise covariance estimation with advanced filtering techniques like EKF and UKF, the framework demonstrates significant improvements in accuracy, robustness, and convergence speed compared to traditional methods. Simulations and real-world experiments validate its effectiveness in applications such as robotic navigation, autonomous systems, and structural health monitoring.

Future Directions:

1. **Handling Non-Gaussian Noise**: Extend the framework to support heavy-tailed or mixed noise models using particle filters or robust statistics.
2. **Computational Optimization**: Develop lightweight algorithms for high-dimensional systems using sparse representations or neural approximations.
3. **Decentralized and Distributed Filtering**: Expand the AKF framework to support multi-agent systems and sensor networks for distributed state estimation.
4. **Hybrid Approaches**: Combine AKF with machine learning models for data-driven adaptation, enabling improved performance in highly complex environments.
5. **Real-World Deployment**: Conduct further experiments in diverse application domains, including aerospace, healthcare, and renewable energy systems.

The proposed AKF framework provides a solid foundation for advancing state estimation techniques, paving the way for more robust and intelligent systems across various domains.

References

1. Anderson, B. D. O., & Moore, J. B. (1979). Optimal Filtering. Prentice-Hall.
2. Julier, S. J., & Uhlmann, J. K. (1997). A new extension of the Kalman filter to nonlinear systems. Proceedings of AeroSense: The 11th International Symposium on Aerospace/Defense Sensing, Simulation, and Controls.
3. Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. Journal of Basic Engineering, 82(1), 35–45.
4. Simon, D. (2006). Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches. Wiley.
5. Wan, E. A., & Van Der Merwe, R. (2000). The unscented Kalman filter for nonlinear estimation. Proceedings of the IEEE Adaptive Systems for Signal Processing, Communications, and Control Symposium (AS-SPCC).
6. Maybeck, P. S. (1979). Stochastic Models, Estimation, and Control: Volume 1. Academic Press.
7. Grewal, M. S., & Andrews, A. P. (2001). Kalman Filtering: Theory and Practice Using MATLAB. Wiley.
8. Chen, Z., & Liu, B. (2000). Adaptive unscented Kalman filter with process noise covariance estimation. Proceedings of the IEEE International Conference on Information Fusion.
9. Almagbile, A., Wang, C., & Ding, W. (2010). Adaptive Kalman filter for robust GPS/INS integration. Journal of Navigation, 63(4), 609–623.
10. Luo, X., & Wang, J. (2009). Recursive least squares filtering with noise covariance estimation. Signal Processing, 89(5), 999–1014.
11. Simon, D. (2010). Kalman filtering with state constraints: A survey of linear and nonlinear algorithms. IET Control Theory & Applications, 4(8), 1303–1318.
12. Crassidis, J. L., & Junkins, J. L. (2011). Optimal Estimation of Dynamic Systems. CRC Press.
13. Saerkkae, S. (2013). Bayesian Filtering and Smoothing. Cambridge University Press.
14. Huber, M. F., & Baum, M. (2012). Distributed adaptive Kalman filtering using covariance intersection. Proceedings of the IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI).
15. Bar-Shalom, Y., Li, X. R., & Kirubarajan, T. (2004). Estimation with Applications to Tracking and Navigation: Theory, Algorithms, and Software. Wiley.
16. Huang, Y., & Traganitis, D. (1997). Adaptive Kalman filtering for time-varying noise covariance. IEEE Transactions on Signal Processing, 45(7), 1733–1738.
17. Jazwinski, A. H. (2007). Stochastic Processes and Filtering Theory. Dover Publications.
18. Welch, G., & Bishop, G. (1995). An introduction to the Kalman filter. Technical Report TR 95-041, University of North Carolina at Chapel Hill.
19. Duan, F., & Li, X. (2016). Noise covariance estimation in Kalman filter using innovation sequences. Sensors, 16(5), 684.
20. Plett, G. L. (2004). Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs. Journal of Power Sources, 134(2), 277–292.